Analyse

Exam

22nd of January of 2007

1 Is $X = \{\frac{1}{n} \mid n \in \mathbb{N}\} \cup \{0\}$ and open subset of $\mathbb{R}$? is it close? is it compact? (give an appropriate argument). (2 points.)

2 Let f and g be two functions from $\mathbb{R}^2$ to $\mathbb{R}$.

(i) Suppose that f and g are continuous at $(0,0)$. Prove that $f+g$ is also continuous at $(0,0)$.

(ii) Suppose f and g are differentiable at $(0,0)$. Prove that $f+g$ is also differentiable at $(0,0)$.

(2 points)

3 Consider $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = \frac{yx^2}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

(i) Prove that f is continuous at $(0,0)$.

(ii) Compute $D_u f(0,0)$ for all $u \in \mathbb{R}^2$.

(iii) Is f differentiable at $(0,0)$?. (give an appropriate argument.)

(2 points)

4 Prove using only the definition of integrability that $f : [0,1] \times [0,1] \longrightarrow \mathbb{R}$ given by $f(x,y) = xy$ is integrable. (2 points)

5 Consider $D = \{(x,y,z) \in \mathbb{R}^3 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\}$ with $a,b,c > 0$. Compute the volume of D. (1 point)